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Neighbourhood V_4- Magic Labeling of Some Splitting Graphs

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Abstract-The Klein 4-group,denoted by V_4 is an abelian group of order 4. It has elements $V_4 = \{0, a, b, c\}$, with $a + a = b + b = c + c = 0$ and $a + b = c, b + c = a, c + a = b$. A graph $G(V(G), E(G)$ is said to be neighbourhood V_4- magic if there exists a labeling $f : V(G) \rightarrow V_4\backslash\{0\}$ such that the induced mapping $N_f^+ : V(G) \rightarrow V_4$ defined by $N_f^+(v) = \sum_{u \in N(v)} f(u)$ is a constant map. If this constant is $p(p \neq 0)$, we say that f is a $p-neighbourhood$ V_4- magic labeling of G and G a $p-neighbourhood$ V_4- magic graph. If this constant is zero, we say that f is a $0-neighghbourhood$ V_4-magic labeling of G and G a $0-neighbourhood$ V_4- magic graph. In this paper, we discuss neighbourhood V_4- magic labeling of some splitting graphs.

Key words: Klein-4-group, Splitting graphs, a-neighbourhood and 0-neighbourhood V_4-magic graphs.
AMS classification: 05C78, 05C25.

1 Introduction

Throughout this paper we consider simple, finite, connected and undirected graphs. For standard terminology and notation we follow [1] and [6]. For a detailed survey on graph labeling we refer [2]. The V_4-magic graphs were introduced by S. M. Lee et al. in 2002 [3]. We say that, a graph $G = (V(G), E(G))$, with vertex set $V(G)$ and edge set $E(G)$ is Neighbourhood V_4-magic if there exists a labeling $f : V(G) \rightarrow V_4\backslash\{0\}$ such that the induced mapping $N_f^+ : V(G) \rightarrow V_4$ defined by $N_f^+(v) = \sum_{u \in N(v)} f(u)$ is a constant map. If this constant is p, where p is any non zero element in V_4, then we say that f is a $p-neighbourhood$ V_4-magic labeling of G and G is said to be a $p-neighbourhood$ V_4-magic graph. If this constant is 0, then we say that f is a $0-neighbourhood$ V_4-magic labeling of G and G is said to be a $0-neighbourhood$ V_4-magic graph. We divide the class of neighbourhood V_4-magic graphs into the following three categories:

(i) $\Omega_a :=$ the class of all $a-neighbourhood$ V_4-magic graphs,

(ii) $\Omega_0 :=$ the class of all $0-neighbourhood$ V_4-magic graphs, and

(iii) $\Omega_{a,0} := \Omega_a \cap \Omega_0$.

The Splitting graph $S(G)$ of a connected graph G is obtained by adding to each vertex u in G, a new vertex u' such that u' is adjacent to the neighbours of u in G. The Bistar $B_{m,n}$ is the graph obtained by joining the central vertex $K_{1,m}$ and $K_{1,n}$ by an edge[2].The friendship graph or the Dutch windmill graph, denoted by F_m (or $D_3^{(m)}$) is obtained by taking m copies of C_3 with one vertex in common[5]. The Book graph B_n is the graph $S_n \square P_2$, where S_n is the star with $n+1$ vertices and P_2 is the path on 2 vertices[7]. A quadrilateral snake QS_n is obtained from a path $v_1v_2v_3 \ldots v_n$ by joining each pair v_i, v_{i+1} to the new vertices u_i, w_i respectively and then joining u_i and w_i by an edge[4]. This paper investigate neighbourhood V_4- magic labeling of splitting graphs of $C_n, P_n, B_{m,n}, K_{1,n}, K_{m,n}, F_m, QS_n$ and B_n.

2 Main Results

Theorem 2.1. *The graph $S(C_n) \in \Omega_a$ if and only if $n \equiv 0(mod\ 4)$.*

Proof. Consider the splitting graph $S(C_n)$, let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of C_n and let $u'_1, u'_2, u'_3, \ldots, u'_n$ be the new vertices in $S(C_n)$. Assume that $n \not\equiv 0(mod\ 4)$. Then either $n \equiv 1(mod\ 4)$ or $n \equiv 2(mod\ 4)$ or $n \equiv 3(mod\ 4)$. We show that in each these cases $S(C_n) \notin \Omega_a$.

Case 1: $n \equiv 1(mod\ 4)$

In this case $n = 4k+1$ for some $k \in \mathbb{N}$. Then $V(S(C_n)) = \{u_i, u'_i : 1 \leq i \leq 4k+1\}$. If possible let $S(C_n) \in \Omega_a$ with a labeling f. Then $N_f^+(u'_2) = a$ implies that $f(u_1) + f(u_3) = a$, which implies that either $f(u_1) = b$ or $f(u_1) = c$. Without loss of generality assume that $f(u_1) = b$. Then $f(u_3) = c, f(u_5) = b, f(u_7) = c, f(u_9) = b, f(u_{11}) = c, f(u_{13}) = b$. Proceeding like this, we get $f(u_{4k+1}) = b$. Also $N_f^+(u'_1) = a$ and $f(u_{4k+1}) = b$ implies that $f(u_2) = c, f(u_4) = b, f(u_6) = c, f(u_8) = b, f(u_{10}) = c, f(u_{12}) = b$. Proceeding like this we get $f(u_{4k}) = b$. Therefore, $N_f^+(u'_{4k+1}) = b+b = 0$, a contradiction. Thus if $n \equiv 1(mod\ 4)$, we have $S(C_n) \notin \Omega_a$.

Case 2: $n \equiv 2(mod\ 4)$

In this case $n = 4k+2$ for some $k \in \mathbb{N}$. Then $V(S(C_n)) = \{u_i, u'_i : 1 \leq i \leq 4k+2\}$. If possible let $S(C_n) \in \Omega_a$ with a labeling f. Then $N_f^+(u'_2) = a$ implies that $f(u_1) + f(u_3) = a$, which implies that either $f(u_1) = b$ or $f(u_1) = c$. Without loss of generality assume that $f(u_1) = b$. Then $f(u_3) = c, f(u_5) = b, f(u_7) = c, f(u_9) = b, f(u_{11}) = c, f(u_{13}) = b$. Proceeding like this, we get $f(u_{4k+1}) = b$. Now $N_f^+(u'_{4k+2}) = f(u_1) + f(u_{4k+1}) = b + b = 0$, a contradiction. Thus if $n \equiv 2(mod\ 4)$, we have $S(C_n) \notin \Omega_a$.

Case 3: $n \equiv 3(mod\ 4)$

In this case $n = 4k+3$ for some $k \in \mathbb{N}$. Then $V(S(C_n)) = \{u_i, u'_i : 1 \leq i \leq 4k+3\}$. If possible let $S(C_n) \in \Omega_a$ with a labeling f. Then $N_f^+(u'_2) = a$ implies that $f(u_1) + f(u_3) = a$, which implies that either $f(u_1) = b$ or $f(u_1) = c$. Without loss of generality assume that $f(u_1) = b$. Then $f(u_3) = c, f(u_5) = b, f(u_7) = c, f(u_9) = b, f(u_{11}) = c, f(u_{13}) = b$. Proceeding like this, we get $f(u_{4k+3}) = c$. Now $N_f^+(u'_1) = a$ implies that $f(u_2) = b, f(u_4) = c, f(u_6) = b, f(u_8) = c, f(u_{10}) = b, f(u_{12}) = c$. Proceeding like this, we get $f(u_{4k+2}) = b$. Therefore $N_f^+(u'_{4k+3}) = f(u_1) + f(u_{4k+2}) = b + b = 0$, a contradiction.Thus if $n \equiv 3(mod\ 4)$, we also have $S(C_n) \notin \Omega_a$. Hence if $n \not\equiv 0(mod\ 4)$, $S(C_n) \notin \Omega_a$. Conversely if $n \equiv 0(mod\ 4)$,

Define $f : V(S(C_n)) \to V_4 \backslash \{0\}$ as:

$$f(u_i) = \begin{cases} b & \text{if} \quad i \equiv 1, 2(mod\ 4) \\ c & \text{if} \quad i \equiv 0, 3(mod\ 4) \end{cases}$$

$$f(u'_i) = a \quad \text{for} \quad 1 \leq i \leq n.$$

Then, f is a $a-neighbourhood$ V_4- magic labeling for $S(C_n)$. This completes the proof of the theorem. □

Theorem 2.2. $S(C_n) \in \Omega_0$ *for all* $n \geq 3$.

Proof. The degree of each vertex in $S(C_n)$ is either 2 or 4. By labeling all the vertices by a, we get $N_f^+(u) = 0$ for all $u \in V(S(C_n))$. □

Corollary 2.3. $S(C_n) \in \Omega_{a,0}$ *if and only if* $n \equiv 0(mod\ 4)$.

Proof. Proof is obviously follows from theorem 2.1 and theorem 2.2. □

Theorem 2.4. *The graph* $S(P_n) \notin \Omega_0$ *for all* $n \geq 2$.

Proof. Proof is obvious due to the presence of pendant vertices in $S(P_n)$. □

Theorem 2.5. *$S(P_n) \notin \Omega_a$ for $n \geq 2$.*

Proof. Consider the splitting graph $S(P_n)$, let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of P_n and let $u'_1, u'_2, u'_3, \ldots, u'_n$ be the new vertices in $S(P_n)$. Suppose that $S(P_n) \in \Omega_a$ for some $n \geq 2$ with a labeling f. Then $N_f^+(u'_1) = a$ implies that $f(u_2) = a$. Also $N_f^+(u'_1) = a$ gives $f(u_2) + f(u'_2) = a$, which implies that $f(u'_2) = 0,$ a contradiction. This completes the proof of the theorem. □

Corollary 2.6. *$S(P_n) \notin \Omega_{a,0}$ for $n \geq 2$.*

Proof. Proof directly follows from theorems 2.4 and 2.5. □

Theorem 2.7. *$S(B_{m,n}) \notin \Omega_a$ for all $m > 1$ and $n > 1$.*

Proof. Consider the bistar $B_{m,n}$ with vertex set $V = \{u, v, u_i, v_j : 1 \leq i \leq m$ and $1 \leq j \leq n\}$ where $u_i (1 \leq i \leq m)$ and $v_j (1 \leq j \leq n)$ are pendant vertices adjacent to u and v respectively. Let $V' = \{u', v', u'_i, v'_j : 1 \leq i \leq m$ and $1 \leq j \leq n\}$ be the corresponding set of new vertices in $S(B_{m,n})$. Then $V(S(B_{m,n})) = V \cup V'$. Suppose that $S(B_{m,n}) \in \Omega_a$ for some $m > 1$ and $n > 1$ with a labeling f. Then $N_f^+(v'_1) = a$ implies that $f(v) = a.$ Now $N_f^+(v_1) = a$ gives $f(v) + f(v') = a$, which implies that $f(v') = 0,$ a contradiction. This completes the proof of the theorem. □

Theorem 2.8. *$S(B_{m,n}) \notin \Omega_0$ for all $m > 1$ and $n > 1$.*

Proof. Proof is obvious, since $S(B_{m,n})$ has pendant vertices. □

Corollary 2.9. *$S(B_{m,n}) \notin \Omega_{a,0}$ for all $m > 1$ and $n > 1$.*

Proof. Proof directly follows from theorems 2.7 and 2.8. □

Theorem 2.10. *$S(K_{1,n}) \notin \Omega_a$ for all $n \in \mathbb{N}$.*

Proof. Consider $K_{1,n}$ with vertex set $V = \{u, u_i : 1 \leq i \leq n\}$ and let $V' = \{u', u'_i : 1 \leq i \leq n\}$ be the corresponding set of new vertices in $S(K_{1,n})$. Assume that $S(K_{1,n}) \in \Omega_a$ for some $n \in \mathbb{N}$ with a labeling f. Now $N_f^+(u'_1) = a$ gives $f(u) = a$. Also $N_f^+(u_1) = a$ implies that $f(u) + f(u') = a$, which implies that $f(u') = 0,$ a contradiction. Hence the proof. □

Theorem 2.11. *$S(K_{1,n}) \notin \Omega_0$ for all $n \in \mathbb{N}$.*

Proof. Proof is obvious due to the presence of pendant vertices in $S(K_{1,n})$. □

Corollary 2.12. *$S(K_{1,n}) \notin \Omega_{a,0}$ for all $n \in \mathbb{N}$.*

Proof. Proof directly follows from theorems 2.10 and 2.11. □

Theorem 2.13. *$S(K_{m,n}) \in \Omega_a$ for all $m > 1$ and $n > 1$.*

Proof. Consider $K_{m,n}$ with $m > 1$ and $n > 1$. Let $X = \{u_1, u_2, u_3, \ldots, u_m\}$ and $Y = \{v_1, v_2, v_3, \ldots, v_n\}$ be the bipartition of $K_{m,n}$. Also let $X' = \{u'_1, u'_2, u'_3, \ldots, u'_m\}$ and $Y' = \{v'_1, v'_2, v'_3, \ldots, v'_n\}$ be the corresponding sets of new vertices in $S(K_{m,n})$. Then $V(S(K_{m,n})) = X \cup Y \cup X' \cup Y'$. We consider the following cases:

Case 1: Both m and n are even.

Define $f : V(S(K_{m,n})) \to V_4 \backslash \{0\}$ as:

$$f(u_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(u'_i) = a \text{ for } 1 \leq i \leq m$$

$$f(v'_j) = a \text{ for } 1 \leq j \leq n$$

Case 2: m is even and n is odd.

$$f(u_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v'_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(u'_i) = a \text{ for } 1 \le i \le m$$

$$f(v_j) = a \text{ for } 1 \le j \le n$$

Case 3: m is odd and n is even.

$$f(u'_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(u_i) = a \text{ for } 1 \le i \le m$$

$$f(v'_j) = a \text{ for } 1 \le j \le n$$

Case 4: Both m and n are odd.

$$f(u'_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v'_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(u_i) = a \text{ for } 1 \le i \le m$$

$$f(v_j) = a \text{ for } 1 \le j \le n$$

In each of the above cases, f is a a-neighbourhood V_4-magic labeng of $S(K_{m,n})$. This completes the proof of the theorem. □

Theorem 2.14. *$S(K_{m,n}) \in \Omega_0$ for all $m > 1$ and $n > 1$.*

Proof. Consider $K_{m,n}$ with $m > 1$ and $n > 1$. Let $X = \{u_1, u_2, u_3, \ldots, u_m\}$ and $Y = \{v_1, v_2, v_3, \ldots, v_n\}$ be the bipartition of $K_{m,n}$. Also let $X' = \{u'_1, u'_2, u'_3, \ldots, u'_m\}$ and $Y' = \{v'_1, v'_2, v'_3, \ldots, v'_n\}$ be the corresponding sets of new vertices in $S(K_{m,n})$. Then $V(S(K_{m,n})) = X \cup Y \cup X' \cup Y'$. We consider the following cases:

Case 1: Both m and n are even.

Define $f : V(S(K_{m,n})) \to V_4 \backslash \{0\}$ as:

$$f(u_i) = f(u'_i) = a \quad \text{if} \quad i = 1, 2, 3, \ldots, m$$

$$f(v_j) = f(v'_j) = a \quad \text{if} \quad j = 1, 2, 3, \ldots, n$$

Case 2: m is even and n is odd.

$$f(v_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(v'_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(u_i) = a \text{ for } 1 \le i \le m$$

$$f(u'_i) = a \text{ for } 1 \le i \le m$$

Case 3: m is odd and n is even.

$$f(u_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(u'_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v_j) = a \text{ for } 1 \le j \le n$$

$$f(v'_j) = a \text{ for } 1 \le j \le n$$

Case 4: Both m and n are odd.

$$f(u_i) = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(u_i') = \begin{cases} b & \text{if} \quad i = 1 \\ c & \text{if} \quad i = 2 \\ a & \text{if} \quad i > 2 \end{cases}$$

$$f(v_j) = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

$$f(v_j') = \begin{cases} b & \text{if} \quad j = 1 \\ c & \text{if} \quad j = 2 \\ a & \text{if} \quad j > 2 \end{cases}$$

In each of the above cases, f is a 0-neighbourhood V_4-magic labeng of $S(K_{m,n})$. This completes the proof of the theorem. ☐

Corollary 2.15. *$S(K_{m,n}) \in \Omega_{a,0}$ for all $m > 1$ and $n > 1$.*

Proof. Proof directly follows from theorems 2.13 and 2.14. ☐

Theorem 2.16. *$S(F_m) \in \Omega_0$ for all $m \in \mathbb{N}$.*

Proof. If we label all the vertices of $S(F_m)$ by a, we get $S(F_m) \in \Omega_0$. ☐

Theorem 2.17. *$S(F_m) \notin \Omega_a$ for all $m \in \mathbb{N}$.*

Proof. Consider the friendship graph F_m. Let the vertices of i^{th} copy of C_3 in F_m be w, u_i and v_i where w is the common vertex of the triangles and let $\{w', u_i', v_i' : 1 \leq i \leq m\}$ be the corresponding set of vertices in $S(F_m)$. Assume that $S(F_m) \in \Omega_a$ for some $m \in \mathbb{N}$ with a labeling f. Since $N_f^+(u_1') = a$, either $f(w) = b$ or $f(w) = c$. Without loss of generality assume that $f(w) = b$. If $f(w) = b$, $f(u_i) = f(v_i) = c$ for all $1 \leq i \leq m$. Therefore, $N_f^+(w') = 2mc = 0$, a contradiction. Hence $S(F_m) \notin \Omega_a$ for all $m \in \mathbb{N}$. ☐

Corollary 2.18. *$S(F_m) \notin \Omega_{a,0}$ for all $m \in \mathbb{N}$.*

Proof. Proof directly follows from theorems 2.16 and 2.17. ☐

Theorem 2.19. *$S(QS_n) \in \Omega_0$ for all $n \in \mathbb{N}$.*

Proof. Label all the vertices of $S(QS_n)$ by a, we get $S(QS_n) \in \Omega_0$. ☐

Theorem 2.20. *$S(QS_n) \notin \Omega_a$ for all $n > 2$.*

Proof. Let QS_n be the quadrilateral snake obtained from the path $v_1 v_2 v_3 \ldots v_n$ by joining each pair v_i, v_{i+1} to the new vertices u_i, w_i respectively and then joining u_i and w_i by an edge. Now consider $S(QS_n)$. Let v_i', u_i', w_i' be the new vertices corresponding to v_i, u_i, w_i. Suppose $S(QS_n) \in \Omega_a$ for some $n > 2$ with a labeling f. Then, $N_f^+(u_1') = a$ gives $f(v_1) + f(w_1) = a$. Also $N_f^+(w_2') = a$ implies that $f(u_2) + f(v_3) = a$, Therefore, $N_f^+(v_2') = f(v_1) + f(w_1) + f(u_2) + f(v_3) = 0$, a contradiction. Hence, $S(QS_n) \notin \Omega_a$ for all $n > 2$. ☐

Corollary 2.21. *$S(QS_n) \notin \Omega_{a,0}$ for all $n > 2$.*

Proof. Proof directly follows from theorems 2.19 and 2.20. ☐

Theorem 2.22. *$S(B_n) \in \Omega_a$ if and only if n is odd.*

Proof.
Consider B_n with vertex set $\{u, v, u_i, v_i : 1 \leq i \leq n\}$ and edge set $\{uv, uu_i, vv_i, u_i v_i : 1 \leq i \leq n\}$. Let $\{u', v', u_i', v_i' : 1 \leq i \leq n\}$ be the set of new vertices in $S(B_n)$. Assume that $S(B_n) \in \Omega_a$ for some $n \in \mathbb{N}$ with a labeling f. Since $N_f^+(u_1') = a$, we have $f(u) = b$ or $f(u) = c$. Without loss of generality we assume that $f(u) = b$. Then $f(v_i) = c$ for all $i = 1, 2, 3, \ldots, n$. Now $N_f^+(v') = a$ implies that $f(u) + \sum_{i=1}^{n} f(v_i) = b + nc = a$. Hence n is odd. Conversely assume that n is odd. Define a labeling $f : V(S(B_n)) \to V_4 \backslash \{0\}$ as:

$$f(u) = f(u_i) = b \quad \text{if} \quad i = 1, 2, 3, \ldots, n$$

$$f(v) = f(v_i) = c \quad \text{if} \quad i = 1, 2, 3, \ldots, n$$

$$f(u') = fu_i') = a \quad \text{if} \quad i = 1, 2, 3, \ldots, n$$

$$f(v) = f(v_i') = a \quad \text{if} \quad i = 1, 2, 3, \ldots, n$$

Then, f is a $a-$ neighbourhood V_4-magic labeling of $S(B_n)$. This completes the proof. ☐

Theorem 2.23. *$S(B_n) \in \Omega_0$ if and only if n is odd.*

Proof. Consider the book graph B_n with vertex set $\{u, v, u_i, v_i : 1 \leq i \leq n\}$ and edge set $\{uv, uu_i, vv_i, u_iv_i : 1 \leq i \leq n\}$. Let $\{u', v', u'_i, v'_i : 1 \leq i \leq n\}$ be the set of new vertices in $S(B_n)$. Assume that $S(B_n) \in \Omega_0$ for some $n \in \mathbb{N}$ with a labeling f. Since $N_f^+(u'_1) = 0,$ we should have $f(u) = f(v_1) = a$ or $f(u) = f(v_1) = b$ or $f(u) = f(v_1) = c.$ Without loss of generality we assume that $f(u) = f(v_1) = a.$ Then $f(v_i) = a$ for all $i = 1, 2, 3, \ldots, n.$ Now $N_f^+(v') = 0$ implies that $f(u) + \sum_{i=1}^{n} f(v_i) = a + na = 0.$ Hence n is odd. Conversely assume that n is odd. We define $f : V(S(B_n)) \to V_4 \backslash \{0\}$ as: $f(w) = a$ for all $w \in V(S(B_n)).$ Then, f is a $0-$ neighbourhood V_4-magic labeling of $S(B_n)$. □

Corollary 2.24. *$S(B_n) \in \Omega_{a,0}$ if and only if n is odd.*

Proof. Proof directly follows from theorems 2.22 and 2.23. □

This research paper investigates neighbourhood V_4- magic labeling of splitting graphs of special graphs like $C_n, P_n,$ $B_{m,n},$ $K_{1,n},$ $K_{m,n},$ F_m, QS_n and B_n respectively. The splitting graph we considered was 2-level splits. The $m-$ level splitting graphs $(m > 2)$ yet to be considered. Scope of this research is the investigation of neighbourhood V_4-magic labeling of $m-$ level splitting graphs for $m > 2$.

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